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Dynamic Policy Games in Economics
F. van der Ploeg and A.J. de Zeeuw, (Editors)

MACROECONOMIC POLICY GAMES WITH INCOMPLETE INFORMATION: SOME EXTENSIONS*

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Introduction

In recent work on the inflationary bias in macroeconomic policy (Backus and Driffill 1985), the choice of policy has been modelled as a game between the private sector of the economy and the government, in which the private sector has incomplete information about the government's preferences. The private sector does not know whether the government is committed to a non-inflationary policy or whether it is maximising an objective function which puts negative weight on inflation and positive weight on expanding output above the natural level. The macroeconomic model which is used is a static and deterministic one which embodies the 'price-surprise' model of output in its simplest form: in each period the excess of output above the natural rate is proportional to the inflation surprise. The only source of uncertainty in the model arises from the randomising strategies which the government may choose. The government is assumed to choose the actual rate of inflation in each period, and the private sector 'chooses' the expected rate of inflation.

The analysis of this as a finitely repeated game, drawing heavily on the work of Kreps and Wilson (1982), produces a sequential equilibrium in which the non-committed government imitates the committed government until the end of the game draws close. In each period of the game, the government has a 'reputation': the probability with which the private sector agents believe that the government is committed to zero inflation; and this belief is updated each period by Bayes' rule according to the event observed (inflation/no inflation) and the strategies which each possible type of government would adopt at that stage of the game. Since a committed government is known never to inflate, and since the private sector perfectly monitors the government's actions, the observation of inflation immediately reduces the government's reputation to zero. The cost associated with losing one's reputation in this way is sufficiently great to deter the non-committed government from causing inflation, until the end of the game approaches. As the game draws near its end, the government may become indifferent between inflating and not inflating, in which case it may pursue a randomising strategy, mixing the strategies (inflate/do not inflate). Conditional on the realisation of no inflation when randomising, its reputation in the next period will be higher. In the papers by Backus and Driffill (1985a) and Barro (1986) the increase in reputation which occurs is just sufficient to offset the effects of the increased tendency of the non-committed government to inflate as time goes by, so that overall the expected inflation rate (or its mean value) remains constant during the period of time when the government is randomising.

The main result from this analysis is that the uncertainty in the minds of the private sector is sufficient to induce the uncommitted government to adhere to the policy of the committed government for most of the game. No formal constitutional amendment or restriction on government freedom of action is needed in order to achieve the socially efficient outcome, except near the end of the game.

The results of those papers may be sensitive to some of the restrictive assumptions made about the beliefs and information of private sector agents and the strategies which are open to the government. For example, Vickers (1986) modifies the structure of the game slightly by introducing two different types of government that each give some weight to employment in their objective function, but differ in the amount. This contrasts with the assumption of an uncommitted and a committed government used by Barro (1986) and by Backus and Driffill (1985a). Comparison of this analysis with Backus and Driffill shows that the assumed 'committed' player simplifies the problem and makes it easier to get a pooling equilibrium rather than a separating equilibrium. The player who is committed to zero inflation always wants to play zero under any circumstances whatever, and consequently any other action immediately identifies its perpetrator as being uncommitted. The uncommitted player then has effectively two choices of action in each period of time: (i) zero inflation and (ii) his best response to the public's expectation. The scope for pooling exists, because the committed player has no interest in the action an uncommitted player would take and does not attempt to take an action which would distinguish him from an uncommitted player. By contrast, in Vickers' formulation, the player who cares less about unemployment has an incentive to take actions which separate him from the other type of player. Vickers shows that pooling equilibria can in a wide class of situations be ruled out. In a two-period game, the two types of players take different actions in the first period so that the type is completely revealed in the second period. In the first period, the less concerned (about unemployment) player chooses an inflation rate which is so low that a more concerned player would not wish to imitate his behaviour. This result contrasts sharply with the pooling equilibrium of the Backus and Driffill formulation. It suggests that one should investigate the way alternative assumptions about private agents' priors affect the solutions of the games.

In Section 1 below the analysis of Vickers is extended to include cases where pooling equilibria are possible. In addition, in Section 2, a situation in which private agents believe that governments may come in more than two types is considered. In this case, separating and pooling equilibria are possible, depending on how similar the possible government types are.

One stark result of the game in which private agents can monitor perfectly the actions of the government, and where there is no uncertainty in the execution of policy actions is that, if the inflation rate ever deviates from zero, the government's reputation immediately and irrevocably falls to zero. This result is too strong for the real world, where governments have only indirect control over the rate of inflation, many random events intervene its determination, and the private sector may not be able to monitor the government's actions independently of the final outcome. Thus it would be interesting to examine a model which embodies these factors.

In Section 3 of the paper this is attempted. It is assumed that the government is either committed to zero inflation or uncommitted, as before. A government is assumed to take one of two actions with respect to inflation -intend zero inflation, or intend inflation- and the actual outcome is not certain to be the one intended. There is a chance that the opposite of the intended outcome may occur. The private sector can observe only the final outcome, the actual inflation rate, and cannot tell whether or not it was intended.

In this situation, application of Bayes' rule to the evolution of reputation shows that the observation of inflation does not immediately reduce reputation to zero, since it can always occur even if the government is committed. In a two-period game it now results that the uncommitted government will always play either a pure strategy (intend inflation) or a mixed strategy in the first period, such that it is indifferent between inflating and not inflating. Its reputation improves if no inflation occurs and deteriorates if some inflation occurs.

It appears that a government with a moderately good reputation is likely to put more probability on intending zero inflation than is a government with either a very good or a very poor reputation, since near the extremes it is not possible for the government to influence its reputation much and so the cost of inflating is lower than for middling reputations. It further appears that even if the probability of error is small, so that actual inflation is very likely to equal intended inflation, then the committed government will not pursue the pure strategy of intending zero inflation, but will always mix to some extent. This shows that the ability to get close to the efficient outcome depends on the information and belief structure to a considerable degree.

The analysis of this model is carried out for a two-period game. Even for the very simple structure proposed here, the analysis becomes so complicated as to prevent general results for a T-period game (T greater than 2) from being derived.

The model discussed in Section 3 is related to work by Canzoneri (1985) and by Söderström (1985) in that it includes, as they have done, imperfect monitoring of government actions. Söderström analyses a game between a government and a centralised labour union in which the government monitors the union's action with error, but is fully informed about the union's objectives. As here, only two outcomes are possible for each player. The union sets wages, and can either be aggressive or passive. The government sets monetary policy, which can be either accommodatory or non-accommodatory. Söderström examines an infinitely repeated game and uses a solution proposed by Rubinstein (1979), which induces the union to play a passive wage strategy in each period of time. The government 'punishes' the union by not accommodating wage increases if they have occurred frequently enough in the past. The critical frequency of occurrence is a diminishing function of the duration of the game, and approaches the frequency of naturally occurring (exogenous) wage shocks as the game proceeds. This result suggests imperfect monitoring alone is not enough to cause inefficient government behaviour.

Canzoneri (1985) analyses a somewhat different model where the government has private and incommunicable information about money demand. In his model the money stock (government's control variable) can vary continuously rather than taking on just two discrete values. The private sector responds by anticipating inflation for a 'reversionary period' if it exceeds some critical value in a period. This happens randomly, and thus induces occasional apparent breakdowns of cooperation between the private sector and the government. Meanwhile, the government is discouraged from causing inflation in each period, a result which parallels that of Söderström.

Section 4 of the paper contains some conclusions.

1. Differing Tastes: Two Types, Many Strategies

In this part of the paper the structure used by Vickers (1986) is adopted. Consider a two-period game played between the private sector and the government. In each period the government chooses an inflation rate x_{T-1}, x_T; and the private sector forms expectations of that period's inflation rate x^e_{T-1}, x^e_T. There are two potential governments which differ in the weight they put on output in their objective functions. The per period payoff for government of type i is

$$u_i = -x_t^2/2 + c_i(x_t - x_t^e), \tag{1.1}$$

where t = T-1, T and $0 < c_1 < c_2$. Thus the game runs for periods T-1 and T. Both types of government care somewhat about output, but type 2 cares more than type 1. This is basically the same set-up as Backus and Driffill (1985a) and Barro (1986), except the taste difference replaces the committed/uncommitted distinction, and we allow any action on the real line, rather than a choice from just two. As in those models, the private sector is assumed to attach an initial probability to the event that the government is of type 1, p_{T-1}, at the start of period T-1, which it updates as the game proceeds. The government knows its own type. We look for a sequential equilibrium of the game.

In the final period, T, the government takes x_T^e as being fixed and clearly sets $x_T = c_i$.

Thus the payoff in the final period for a government of type i is

$$u_i(T, x_T^e) = c_i^2/2 - c_i x_T^e.$$

Thus the total payoff for government of type i over both periods of the game, given that in T-1 the private sector expects inflation x_{T-1}^e is

$$U_i = -x_{T-1}^2/2 + c_i(x_{T-1} - x_{T-1}^e) + c_i^2/2 - c_i x_T^e. \tag{1.2}$$

The real action in this game comes in the first period (T-1) when the government chooses an inflation rate x_{T-1} taking x_{T-1}^e as given, but taking into account the way in which the private sector forms its expectations in period T as a function of the inflation rate observed in T-1.

U_i will be used to denote the utility of government of type i which takes action x_{T-1} in period T-1, and c_i in T, when the private sector forms expectations x_T^e of inflation in T (x_{T-1}^e is taken as some fixed number). Effectively we are considering utility effects of action taken after x_{T-1}^e has been chosen. Thus only changes in x_{T-1} and x_T^e affect the value of $U_i(x_{T-1}, x_T^e)$. Note that the value of x_{T-1}^e does not affect the ranking over (x_{T-1}, x_T^e) combinations for any player.

The private sector will form expectations which are rational, and since there are two discrete government types, they can use a step function, such that $x_T^e = x_1$ if $x_{T-1} \le x^*$ and $x_T^e = x_2$ if $x_{T-1} > x^*$, for some values of x_1, x_2 and x^*.

Two possibilities are (i) a separating equilibrium and (ii) a pooling equilibrium. In the separating equilibrium, following Vickers, we have expectations formed by

$$x_T^e = \begin{bmatrix} c_1 \text{ if } x_{T-1} \le k_B \\ \\ c_2 \text{ if } x_{T-1} > k_B \end{bmatrix} \tag{1.3a}$$

where k_B is defined by

$$U_2(k_B, c_1) = U_2(c_2, c_2) \tag{1.3}$$

and that the type-2 government is indifferent between inflating at rate c_2 and being identified as type 2 on the one hand, and choosing low inflation k_B and being identified as of type 1. We assume that the type-2 government then chooses inflation rate c_2 in this case. At the same time, the type-1 government must find it in its interest to choose the lower rate of inflation k_B in T-1, and thus be identified as of type 1. The condition

$$U_1(k_B, c_1) > U_1(c_1, c_2) \tag{1.4}$$

must therefore be satisfied.

A separating equilibrium is illustrated in Figure 1, which follows Vickers (1986). The indifference curves of a government of type i in (x_{T-1}, x_T^e) space are parabolae which peak at $x_{T-1} = c_i$. The slope of the indifference curves of U_i defined by (1.2) above is

$$\left.\frac{dx_T^e}{dx_{T-1}}\right|_{U_i \text{ const}} = 1 - x_{T-1}/c_i. \tag{1.5}$$

so they all have a slope equal to unity at $x_{T-1} = 0$. In the figure, U_2 is the indifference curve of the type-2 player through (c_2, c_2). It cuts the vertical axis at $(0, c_2/2)$. Thus for $c_1 > c_2/2$ we have a separating equilibrium in the positive quadrant. k_B is defined as above by (1.3). For $c_1 > c_2/2$, (1.4) is automatically satisfied.

Now consider a pooling equilibrium. In this case expectations are formed as follows

$$x_T^e = \begin{bmatrix} \bar{c}, & x_{T-1} \le \ell \\ c_2, & x_{T-1} > \ell \end{bmatrix} \tag{1.6}$$

where $\bar{c} = pc_1 + (1-p)c_2$
and both types of government play $x_{T-1} = \bar{c}$. For this to hold in equilibrium, we have

$$U_2(\ell,\bar{c}) \geq U_2(c_2,c_2) \tag{1.7}$$

and

$$U_1(\ell,\bar{c}) \geq U_1(c_1,c_2). \tag{1.8}$$

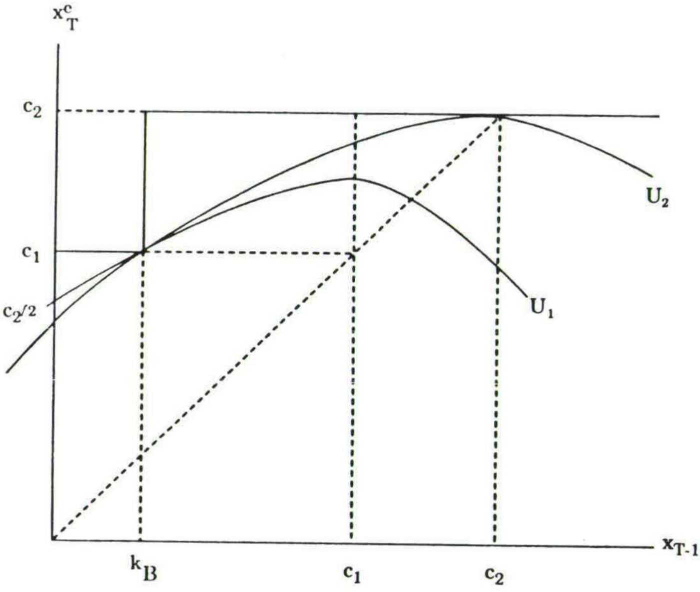

Figure 1: Separating equilibrium

A pooling equilibrium is illustrated in Figure 2 below. It is illustrated for $c_1 > c_2/2$. ℓ lies below c_1, such that both players prefer $(\ell,\bar{c})$ to any other outcome.

A pooling equilibrium does not always exist since there does not always exist an ℓ which satisfies (1.7) and (1.8) above, for $\ell \leq c_1$. As is clear from Figure 2, if $\bar{c}$ is sufficiently close to c_2 because p is sufficiently small, then no $\ell \leq c_1$ may exist such that the type-2 government prefers $(\ell,\bar{c})$ to (c_2,c_2).

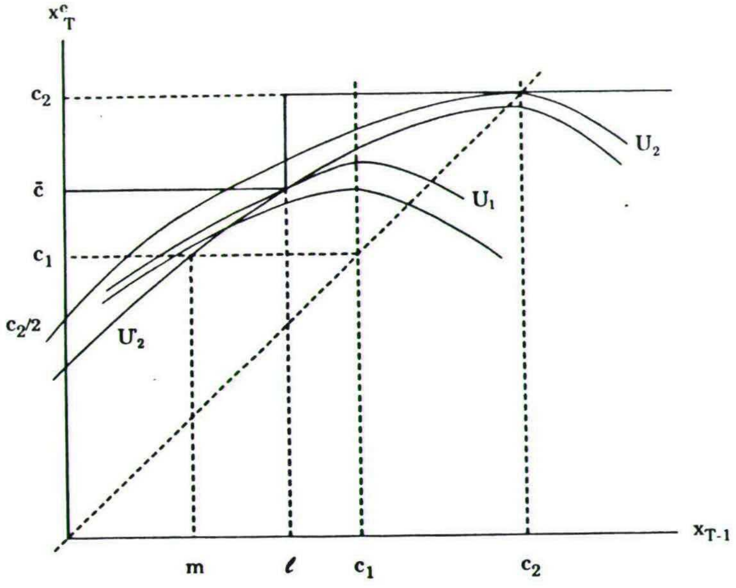

Figure 2: Pooling equilibrium

Vickers rejects pooling equilibria because it is always possible for the type-1 government to defect by choosing a low rate of inflation m such that, if the government could play m and thereby convince the public that it was of type 1, and have the public expect $x_T^e = c_1$, only a type-1 player would find it advantageous to do so. Thus m lies just to the left of the type-2 indifference curve through $(\ell, \bar{c})$. Thus we have

$$U_2(\ell, \bar{c}) > U_2(m, c_1), \tag{1.9}$$

but

$$U_1(\ell, \bar{c}) < U_1(m, c_1). \tag{1.10}$$

Since in the positive orthant the indifference curves of the type-1 government have smaller slopes than those of the type-2 government, such an m always exists, and the pooling equilibrium can always be rejected.

Thus for $c_2 \geq c_1 \geq c_2/2$ a separating equilibrium exists as defined above in (1.3) and (1.4), and no pooling equilibria exist.

For $c_1 < c_2/2$, however, things look different because we can in fact find pooling equilibria which do not fall victim to Vickers' defection argument (above) for rejection, and in some circumstances these appear to dominate the separating equilibrium.

Consider the following equilibria. For $c_1 < c_2/2$ and $\bar{c} > c_2/2$ a pooling equilibrium with mixing exists. The type-2 government plays $x_{T-1} = 0$ with probability q and $x_{T-1} = c_2$ with probability (1-q). The type-1 government plays $x_{T-1} = 0$ certainly. Contingent on $x_{T-1} = 0$ having been observed the probability of the government being of type 1 is

$$\hat{p} = \frac{p}{p + (1-p)q} \tag{1.11}$$

and the expected inflation rate in T is therefore $\hat{p}c_1 + (1-\hat{p})c_2$. q is chosen so that this expected inflation rate is $c_2/2$, i.e. so that

$$\hat{p}c_1 + (1-\hat{p})c_2 = c_2/2.$$

The private sector forms expectations

$$x_T^e = \begin{cases} c_2/2 & \text{if } x_{T-1} \leq 0 \\ c_2 & \text{if } x_{T-1} > 0. \end{cases}$$

Given these expectations, the type-1 government prefers the outcome $(0, c_2/2)$ and the type-2 government is indifferent between $(0, c_2/2)$ and (c_2, c_2) and chooses a random strategy as described above.

This pooling equilibrium is proof against the defection argument since the indifference curve of the type-1 player through $(0, c_2/2)$ is below that of the type-2 player which passes through the same point, except at $x_{T-1} = 0$ where they coincide, as Figure 3 illustrates.

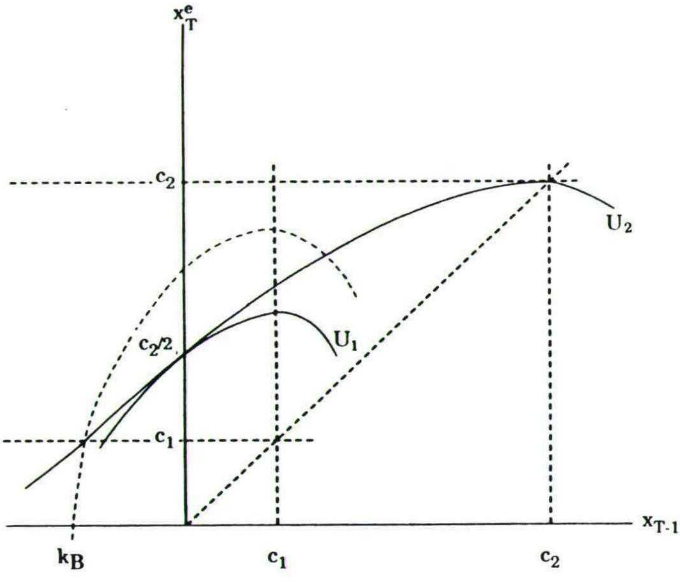

Figure 3: Pooling equilibrium with mixing when $c_1 < c_2/2$ and $\bar{c} > c_2/2$

For the case where $c_1 \le \bar{c} < c_2/2$ an equilibrium exists similar to the last one but in pure strategies. Expectations are formed by

$$x^e_T = \begin{cases} \bar{c} & \text{if } x_{T-1} \le \ell \\ c_2 & \text{if } x_{T-1} > \ell. \end{cases} \tag{1.12}$$

For the type-1 governments

$$U_1(\ell, \bar{c}) \ge U_1(c_1, c_2) \tag{1.13}$$

and, for the type-2 governments,

$$U_2(\ell, \bar{c}) \ge U_2(c_2, c_2), \tag{1.14}$$

where $0 \leq \ell \leq c_1$. In order to prevent defection by type-1 governments, a third condition must be satisfied. There must be no value of x_{T-1} which for $x_T^e = c_1$ is preferred to the pooling outcome $(\ell, \bar{c})$ by type 1 but not by type 2. Thus if j is defined by

$$U_2(j_2, c_1) = U_2(\ell, \bar{c}), \tag{1.15}$$

we have

$$U_1(j, c_1) \leq U_1(\ell, \bar{c}) \tag{1.16}$$

These two equations place an upper limit on the value of ℓ. Note that j is necessarily negative, as Figure 4 illustrates, because the indifference curves of type 1 are flatter than those of type 2 to the right of the vertical axis, and steeper to the left of the vertical axis.

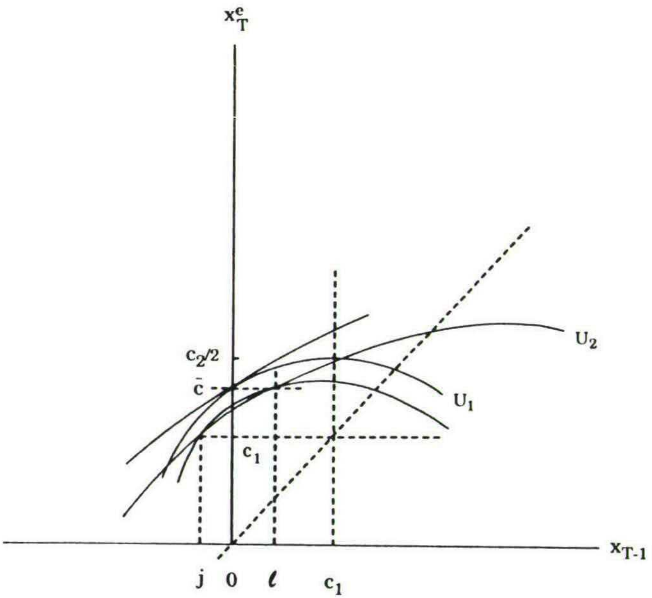

Figure 4: Pooling equilibrium in pure strategies with $c_1 < c_2/2$ and $\bar{c} < c_2/2$

From (1.15) and (1.16) it can be deduced that

$$\ell \le (\bar{c}-c_1)/2$$

or

$$\ell \le (1-p)(c_2-c_1)/2. \tag{1.17}$$

In addition, we have the condition that $\ell \le c_1$.

The higher is the value of ℓ, within the constraints of (1.17) and $\ell \le c_1$, the greater the payoff to each type of government, taking the value of x^e_{T-1} as given. But if initial expectations are taken into account, and noting that $x_{T-1} = x^e_{T-1} = \ell$ for both types of government in this equilibrium, it is clear that both types of players are best off when $\ell = 0$. On grounds of equilibrium dominance, this would therefore be the chosen equilibrium from among these pooling equilibria.

In the Appendix it is shown that in the case where $c_1 \le c_2/2$ and $\bar{c} \le c_2/2$, the pooling equilibrium above dominates the separating equilibrium, where the separating equilibrium exists. All players in the game -governments of both types, and agents in the private sector- prefer the outcome under the pooling equilibrium. In the case where $c_1 > c_2/2$ but $\bar{c} \ge c_2/2$, the pooling equilibrium does not dominate the separating equilibrium, nor is it dominated by the separating equilibrium. Both types of government prefer the pooling equilibrium, but the private sector may for some parameter combinations prefer the separating outcome. Note that the separating equilibrium only exists for $c_1 \ge c_2/9$ in any event.

Consequently, separating equilibria can be ruled out for the first case ($c_1 \le \bar{c} \le c_2/2$) but not for the second.

In summary, the equilibria for a game with two government types can be reduced to the following:

(i) For $c_1 \ge c_2/2$ only a separating equilibrium exists with

$$x^e_T = \begin{cases} c_1 \text{ if } x_{T-1} \le k_B \\ c_2 \text{ if } x_{T-1} > k_B \end{cases}$$

and type-1 governments play $x_{T-1} = k_B$

type-2 governments play $x_{T-1} = c_2$.

(ii) For $c_1 < c_2/2$ but $\bar{c} \geq c_2/2$, a partial pooling equilibrium exists with

$$x_T^e = \begin{cases} c_2/2 & \text{if } x_{T-1} \leq 0 \\ c_2 & \text{if } x_{T-1} > 0 \end{cases}$$

type-1 plays $x_{T-1} = 0$
type-2 plays $x_{T-1} = 0$ with probability q and
$x_{T-1} = c_2$ with probability (1-q)
such that $c_2/2 = c_1\hat{p} + (1-\hat{p})c_2$
where $\hat{p} \equiv p/(p+(1-p)q)$.

For some combinations of parameters (c_1, c_2 and p) a separating equilibrium may also exist.

(iii) For $c_1 < \bar{c} < c_2/2$, a pooling equilibrium exists with expectations

$$x_T^e = \begin{cases} \bar{c} & \text{if } x_{T-1} \leq 0 \\ c_2 & \text{if } x_{T-1} > 0. \end{cases}$$

Both type 1 and type 2 play $x_{T-1} = 0$.

The pooling equilibrium proposed for $c_1 < c_2/2$ holds good as $c_1 \to 0$. By contrast the separating equilibrium is only available for $c_1 \geq c_2/9$, since at that point the constraint (1.4) begins to bind, as Vickers points out. Note that as $c_1 \to 0$, the pooling equilibrium proposed is continuous with the equilibrium in Backus and Driffill (1985a) and Barro (1986). In the limit as $c_1 \to 0$ in the above pooling equilibrium, type 2 plays $x_{T-1} = 0$ if $p \geq \frac{1}{2}$ so that $\bar{c} \leq c_2/2$. If $p < \frac{1}{2}$ then type 2 plays $x_{T-1} = 0$ with probability q such that the *ex-post* probability of type 1, contingent on $x_{T-1} = 0$, is $\hat{p} = \frac{1}{2}$.

2. Many Types

This part of the paper considers how the game is affected when private agents believe at the start of the game that the government's preference for employment (reflected in the parameter c_i) is drawn from a distribution which has some bounded support. As before, a two-period game is considered.

Suppose at T-1 the policy-maker could be of any type c_i in some range, $[\underline{c},\bar{c}]$. Can a separating equilibrium be found? As before, it is assumed that the private sector agents form expectations x_T^e based on their observations of actual inflation at time T-1, x_{T-1}. Let the expectation be represented by the function

$$x_T^e = f(x_{T-1}). \tag{2.1}$$

If the equilibrium is separating, then for each type c_i there is an optimal choice of x_{T-1} given f(.) which is different for each c_i. Thus in equilibrium it must be true that

$$f(x_{T-1}(c)) = c \tag{2.2}$$

for all $c \in [\underline{c},\bar{c}]$, since $x_{T-1}(c)$ reveals the agent's true type c.

If the function $f(x_{T-1})$ is such that x_{T-1} minimises cost, then it satisfies an additional condition. Cost for player of type i is, as in (1.2) above

$$U_i = -x_{T-1}^2/2 + c(x_{T-1}-x_{T-1}^e) + c^2/2 - cf(x_{T-1}). \tag{2.3}$$

Providing f(.) is differentiable and continuous, at an optimum the first order condition is satisfied:

$$\partial U_i/\partial x_{T-1} = -x_{T-1} + c - cf'(x_{T-1}) = 0 \tag{2.4}$$

and

$$\partial^2 U_i/\partial x_{T-1}^2 = -1 - cf''(x_{T-1}) < 0. \tag{2.5}$$

So $f(x_{T-1})$ is a function which for all $c \in [\underline{c},\bar{c}]$ satisfies (2.2) and (2.4) and (2.5). Combining (2.2) and (2.4) gives a differential equation in f:

$$\frac{df(x)}{dx} = 1 - \frac{x}{f(x)}. \tag{2.6}$$

Thus f(x) is a function which has a slope of +1 when x = 0 (f(0) > 0), diminishing to 0 at x = f(x). To tie down the constant of integration, we can set $f(x) = \bar{c}$ when x = f(x). This means that the agent with the highest value of $c(\bar{c})$ playes $x_{T-1} = \bar{c}$: it is not in his interests to reduce x_{T-1} to attempt to signal a lower c.

Having used the upper bound on $\bar{c}$ to position f(.), the lower bound on c is determined by the intersection of f(x) with the x-axis. At this point the player $\underline{\underline{c}}$ sets $x_{T-1}(\underline{\underline{c}}) = 0$. This is illustrated in Figure 5.

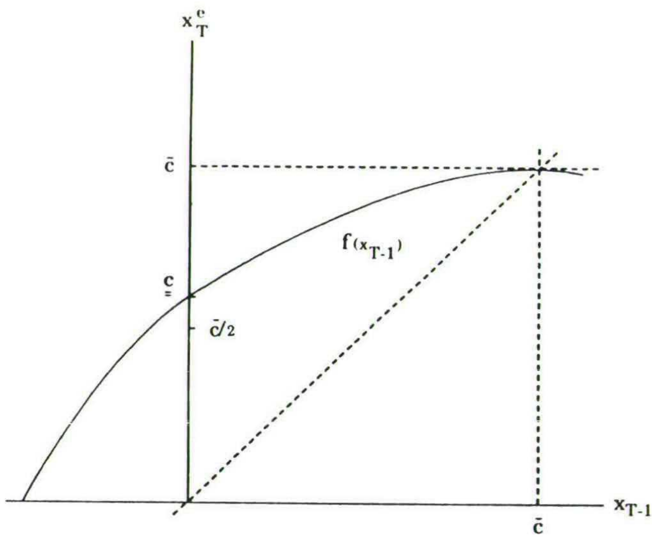

Figure 5: The equilibrium expectations function $x^e_T = f(x_{T-1})$ with a continuum of government types $c \in [\underline{\underline{c}}, \bar{c}]$

The value of c must lie between $\bar{c}$ and $\underline{c}$. If $\underline{c} = \underline{\underline{c}}$, then the player with the lowest value of c sets $x_{T-1} = 0$. This restricts the possible values of c to a narrow range, since $\underline{\underline{c}} > \bar{c}/2$. (This is because the $\bar{c}$ player has an indifference curve passing through $(\bar{c}, \bar{c})$ on the diagram which also passes through $(0, \bar{c}/2)$, and $f(x_{T-1})$ lies above this indifference curve except at $(\bar{c}, \bar{c})$ where they coincide.)

It is not possible to use the part of the function f(.) lying in the NW quadrant where $x \leq 0$ and $f(x) \geq 0$ to induce players with a $c < \underline{\underline{c}}$ to choose a point on that part of the curve, because the second order condition (2.5) is not satisfied on f(x) for $x < 0$. Points of tangency here are maxima of the cost function, not minima. The second order condition (2.5) is satisfied for $c \in [\underline{\underline{c}}, \bar{c}]$ where $x_{T-1}(c) \in [0, \bar{c}]$.

The above procedure defines a value $\underline{\underline{c}}$ as a function of $\bar{c}$. If there were players of type $c < \underline{\underline{c}}$ then the above separating equilibrium would not survive. Governments of type $c < \underline{\underline{c}}$ would choose $x_{T-1} = 0$ and the private sector belief $\underline{\underline{c}} = f(0)$ as defined above would not be rational. In this case it may be possible to find a part pooling/part separating equilibrium.

An equilibrium for this case is postulated as follows. Expectations are formed by the discontinuous function

$$x_T^e = \begin{cases} \hat{c} \text{ if } x_{T-1} = 0 \\ f(x_{T-1}) \text{ if } x_{T-1} \in (\tilde{x}_{T-1}, \bar{c}] \end{cases} \tag{2.7}$$

where

$$\hat{c} = \int_{\bar{c}}^{\tilde{c}} cp(c)dc / \int_{\bar{c}}^{\tilde{c}} p(c)dc \tag{2.8}$$

and

$$\tilde{c} = f(\tilde{x}_{T-1}). \tag{2.9}$$

In this equilibrium, the private sector expects inflation at a rate $\hat{c}$ if it observes $x_{T-1} = 0$, and expects inflation given by the $f(.)$ function derived above for higher inflation in T-1. Governments with $c > \tilde{c}$ prefer to choose an (x_{T-1}, x_T^e) combination on $(x_{T-1}, f(x_{t-1}))$ in the range $x_{T-1} \in (\tilde{x}_{T-1}, \bar{c}]$ rather than to choose $(0, \hat{c})$, and conversely for governments with $c \leq \tilde{c}$.

At the boundary between the two regions is the government of type $\tilde{c}$ that is indifferent between $(0, \hat{c})$ and $(\tilde{x}_{T-1}, f(\tilde{x}_{T-1}))$. Thus, $\tilde{c}$, $\tilde{x}_{T-1}$ and $\hat{c}$ satisfy the condition that

$$U_{\tilde{c}}(0, \hat{c}) = U_{\tilde{c}}(\tilde{x}_{T-1}, f(\tilde{x}_{T-1})) \tag{2.10}$$

where $U_{\tilde{c}}(.)$ is the utility of agent of type $\tilde{c}$ as defined by equation (1.2).

In Figure 6, $\hat{c}$ is plotted on the vertical axis, and against it on the horizontal axis is plotted (i) $\tilde{c}$, the highest value of c such that a government of type c prefers or is indifferent to the combination $(0,\hat{c})$ to any point on the expectations curve $(x_{T-1}, f(x_{T-1}))$, and (ii) the mean value of c from those values less than or equal to $\tilde{c}$.

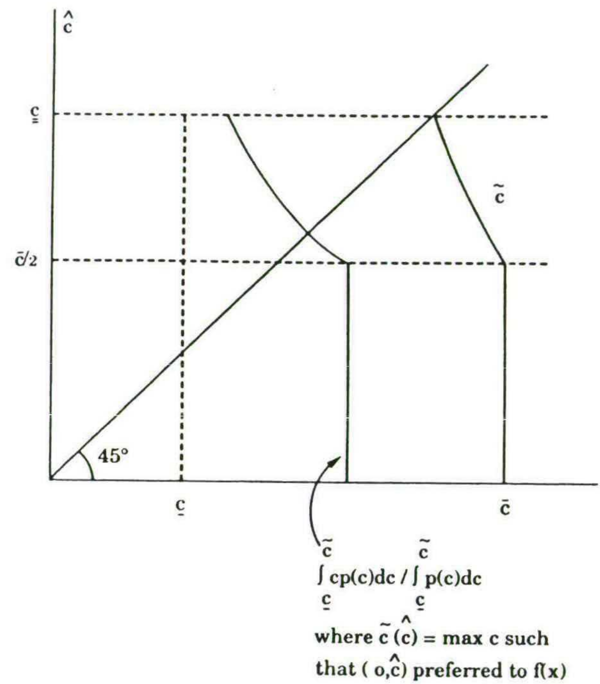

Figure 6: Hybrid pooling/separating equilibrium with a continuum of government types

At $\hat{c} = \underline{\underline{c}}$, all government types with $c \leq \underline{\underline{c}}$ prefer the combination $(0,\hat{c})$ to a point on the curve, and the mean value of c for these types will be less than $\hat{c}$. As $\hat{c}$ is reduced, the value of $\tilde{c}$ rises, until $\hat{c} = \bar{c}/2$ when all types prefer $(0,\hat{c})$.

Thus there will always be a unique value of $\hat{c}$ which satisfies (2.8).

At the equilibrium, all players $c \leq \tilde{c}$ prefer $x_{T-1} = 0$, $x_T^e = \hat{c}$. The average value of c for these players is $\hat{c}$, and consequently private sector expectations are rational. For players $c > \tilde{c}$, a pair $(x_{T-1}, x_T^e = f(x_{T-1}))$ is preferred and they choose a value of x_{T-1} which identifies their true type.

If $\int_{\underline{c}}^{\bar{c}} cp(c)dc \leq \bar{c}/2$, then the average type over all players is less than $\bar{c}/2$, and all players choose to play $x_{T-1} = 0$, $x_T^e = \hat{c}$ where $\hat{c} = \int_{\underline{c}}^{\bar{c}} cp(c)dc$. In this case there is complete pooling.

In Vickers (1986), pooling equilibria are dominated, because some players would have an incentive to defect, play some other x_{T-1}, to persuade the public that they are of a particular type. However, this does not appear to be the case in this instance.

The difference between the result here and Vickers' result is that here the pooling equilibrium occurs with $x_{T-1} = 0$, and in order to achieve separation, a player with a low value of c would have to play a negative inflation rate in T-1, which would give a worse outcome than the pooling equilibrium. However, in Vickers the pooling occurs with a positive inflation rate in T-1, and the low-c player can achieve separation by playing a lower but still positive inflation rate. In that case, the separating deviation pays off better than the pooling equilibrium.

The analysis of this section suggests that the reputational discipline on non-committed governments works when private sector beliefs are more general than in Backus and Driffill (1985). The crucial element is a sufficiently wide disparity in the range of government preferences.

3. Exogenous Uncertainty

This part of the paper examines what happens when the simple model of Backus and Driffill (1985a) is perturbed by the addition of an exogenous random element. The following structure is assumed.

Governments are of two types. Type 1 ('dry') are committed to zero inflation and their preferences in each period could be represented by the function

$$u_{1t} = -x_t^2,$$

where x_t is the rate of inflation in period t. Type 2 ('wet') care about inflation and unemployment and have the following preferences in each period:

$$u_{2t} = -x_t^2/2 + c(x_t - x_t^e),$$

where x_t^e is the private sector's expectation of inflation in period t.

It is assumed as before that only two outcomes for the actual inflation rate are ever observed, either $x_t = 0$ or $x_t = c$. However, it is here assumed that governments do not have perfect control over the actual inflation rate. With some small probability $(1-\pi)$ the government hits the wrong button, so that, if it intends to create inflation equal to c, then c occurs with probability π and zero occurs with probability $(1-\pi)$, and conversely if zero inflation is intended. $\pi > \frac{1}{2}$, so that what is intended is more likely to occur than not. The structure adopted here follows closely that of Söderström (1985) in having two potential outcomes for government behaviour in each period. However, in Söderström, the government's objectives are known for sure. (In his set-up the player who is equivalent to the government here is his trade union.) Another difference between the present set-up and Söderström is that the private sector is assumed to be able to 'choose' any point on the real line when they form expectations, which they do rationally. As before, the game runs until a last period when t = T. The government attempts to maximise the expected value of the sum of the payoffs received up to the end of the game. Thus at time s the government maximises

$$\sum_{t=s}^{T} u_{it}.$$

The private sector enters period t with the belief that with probability p_t the government is of type 1 and with probability $(1-p_t)$ of type 2. Its beliefs are updated using Bayes' rule. Thus if the type-2 player is known to play the strategy of intending zero inflation with probability q_t and intending inflation equal to c with probability $(1-q_t)$, then the private sector updates its beliefs such that

$$p_{t+1}(x_t=0) = \frac{\pi p_t}{\pi p_t + (1-p_t)\tilde{q}_t}, \tag{3.1a}$$

where $\tilde{q} = \pi q_t + (1-q_t)(1-\pi)$, if $x_t = 0$ is observed and

$$p_{t+1}(x_t=c) = \frac{(1-\pi)p_t}{(1-\pi)p_t + (1-p_t)(1-\tilde{q}_t)}, \tag{3.1b}$$

if $x_t = c$ is observed.

The probability of observing inflation equal to c at time t is $(1-\pi)p_t + (1-p_t)(1-\tilde{q}_t)$ and consequently the expected inflation is

$$x_t^e = c[p_t(1-\pi) + (1-p_t)(1-\tilde{q}_t)]. \tag{3.2}$$

Thus the set-up here is exactly the same as in Backus and Driffill (1985a), except that actions are not always carried out as intended due to this element of exogenous uncertainty. This change in the model has significant effects. Providing π is strictly less than one, there is always a positive chance of observing the high inflation outcome, even if the government is actually of type 1 (dry). Consequently, this observation no longer identifies the government as being unmistakably wet and reduces its reputation to zero. Reputation is reduced by a high inflation outcome, but not destroyed. From equation (3.1b) above, it is seen that if $q_t < 1$ so that $\tilde{q}_t < \pi$, and if $\pi < 1$, then conditional on $x_t = c$, $p_{t+1} < p_t$. But if $p_t > 0$, then $p_{t+1} > 0$.

If the realisation is $x_t = 0$, which is always possible regardless of the type of government or strategy played, reputation is not worsened, and improves if the type-2 player is known to play the zero intended inflation strategy with probability less than one ($q_t < 1$).

The greater the amount of noise in the system (i.e. the lower the value of π) the less does the revised reputation differ from the old one, given any initial reputation and strategy choice q_t. In the limit as $\pi_t \to \frac{1}{2}$, the players have no control over the outcome and no observation can change reputation. Of course, there is then no effective difference between the two types of government. Neither has any control at all.

The solution of the game is now considered. In the final period, T, the type-2 player always intends inflation c, and the type-1 player intends zero inflation. The private sector's expectation is

$$x_T^e = c(p_T(1-\pi)+(1-p_T)\pi).$$

The payoff to the wet (type-2) player is

$$-cx_T^e$$

with probability $1-\pi$, and

$$-c^2 + c(c-x_T^e)$$

with probability (π). Hence his expected final period payoff is

$$V_T(p_T) = -c^2(p_T(1-\pi)+(1-p_T)\pi) + \pi c^2/2$$

$$= c^2 p_T(2\pi-1) - \pi c^2/2.$$

Now consider period T-1.

If the wet plays 'intend inflation' equal to zero with probability q_{T-1}, his payoff in T-1 is

$$W_{T-1} = \tilde{q}_{T-1}\{u_2(0,x_{T-1}^e) + V(p_T|x_{T-1} = 0)\}$$

$$+ (1-\tilde{q}_{T-1})\{u_2(c,x_{T-1}^e) + V(p_T|x_{T-1} = c)\}, \tag{3.3}$$

where $u_2(0,x_{T-1}^e) = -cx_{T-1}^e$ and $u_2(c,x_{T-1}^e) = c^2/2 - cx_{T-1}^e$. p_T is determined by equations (3.1a) and (3.1b) above. The government chooses $\tilde{q}_{t-1}$ in the range $\pi \geq \tilde{q}_{t-1} \geq 1-\pi$ to maximise W_{T-1}, taking the private sector's expectations of future inflation as being fixed, since in a sequential equilibrium the strategies (q_{T-1},x_T^e) form a Nash equilibrium pair. Thus the equilibrium values of $\tilde{q}_{T-1}$ satisfy

(i) $\tilde{q}_{T-1} = \pi$ if $u_2(0,x_{T-1}^e) + V(p_T|x_{T-1} = 0)$

$$> u_2(c,x_{T-1}^e) + V(p_T|x_{T-1} = c),$$

(ii) $\pi \geq \tilde{q}_{T-1} \geq 1 - \pi$ if

$$u_2(0,x_{T-1}^e) + V(p_T|x_{T-1} = 0) = u_2(c,x_{T-1}^e) + V(p_T|x_{T-1} = c)$$

for some $\tilde{q}_{T-1} \in [\pi,1-\pi]$.

(iii) $\tilde{q}_{T-1} = 1 - \pi$ if $u_2(0,x^e_{T-1}) + V(p_T|x_{T-1} = 0)$

$$< u_2(c,x^e_{T-1}) + V(p_T|x_{T-1} = c). \tag{3.4}$$

The equilibrium value of $\tilde{q}_{T-1}$ depends on both π and p_{T-1}. It is clear that there is never an equilibrium with $\tilde{q}_{T-1} = \pi$, i.e. with the 'wet' government playing exactly the same strategy as the 'dry' government and intending zero inflation with probability one. In this case p_T would be equal to p_{T-1} regardless of the realisation of x_{T-1}, and the expected payoff from intending zero inflation would be clearly less than from intending inflation equal to c. This result is clearly at variance with the results of Backus and Driffill (1985a) where with a sufficiently good reputation in T-1, the wet player would mimic the dry. The reason for the difference is that the occurrence of high inflation does not now totally ruin the government's reputation. It is no longer the case that $p_T = 0$ if $x_{T-1} = c$.

We now turn to the solution to the wet government's problem at date T-1. From (3.1a) and (3.1b), $p_T(x_{T-1} = 0)$ is a decreasing function of $\tilde{q}_{T-1}$ and $p_T(x_{T-1} = c)$ is an increasing function. $V(T,p_T)$ is a linear increasing function of p_T. If a value $\tilde{q}_{T-1} \in [\pi,1-\pi]$ exists such that equality (3.4) (ii) above is satisfied, then this is the unique solution. If not, then the solution involves $\tilde{q}_{T-1} = (1-\pi)$ and the government prefers the high inflation outcome to the low inflation outcome.

The inequalities (3.4) above reduce to the following

$$(2\pi-1)c^2(p_T(x_{T-1} = 0) - p_T(x_{T-1} = c)) - c^2/2 \begin{cases} > 0; & \tilde{q}_{T-1} = \pi \\ = 0; & \pi \le \tilde{q}_{T-1} \le 1-\pi \\ < 0; & \tilde{q} = (1-\pi). \end{cases} \tag{3.5}$$

It is clear from (3.5) that for the equality to be satisfied we must have $\pi \ge 3/4$. This is a necessary, not sufficient, condition. No wet would ever depart from intending inflation in T-1 if this condition were not satisfied.

It is shown below that for a given value of p_{T-1}, the equilibrium value of q_{T-1} is decreasing in π for $q_{T-1} > 0$, and that for given π, q_{T-1} at first rises with p_{T-1} and then falls. Thus the relationship is as illustrated in Figure 7. In Figure 7b the curve shifts upwards as $\bar{u}$ is increased from 3/4 towards 1. Below some critical value $\underline{p}_{T-1}$, and above a higher critical value $\bar{p}_{T-1}$, the wet government does not randomise but plays the strategy of intending to cause inflation with probability one. At intermediate values of p_{T-1}, randomisation occurs. The reason for this result is that the difference in reputation $p_T(x_{T-1} = 0)$ and $p_T(x_{T-1} = c)$ is wider at intermediate values of p_{T-1} than at extreme values. As $p_{T-1} \to 1$ from below and as $p_{T-1} \to 0$, the difference $p_T(x_{T-1} = 0) - p_T(x_{T-1} = c)$ approaches zero. Because inflating does not injure one's reputation so much if it is either very bad or very good already, there is less disincentive to doing it.

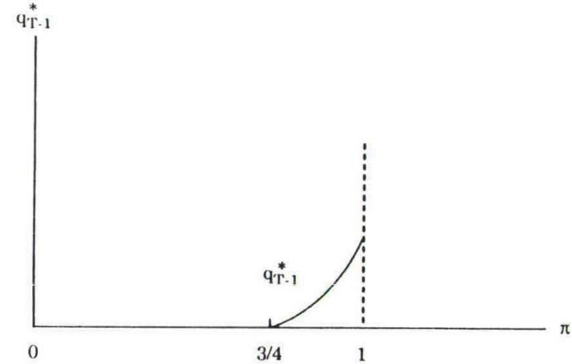

Figure 7a: Relationship between the optimum mixing (q^*_{T-1}) and the probability of error (π)

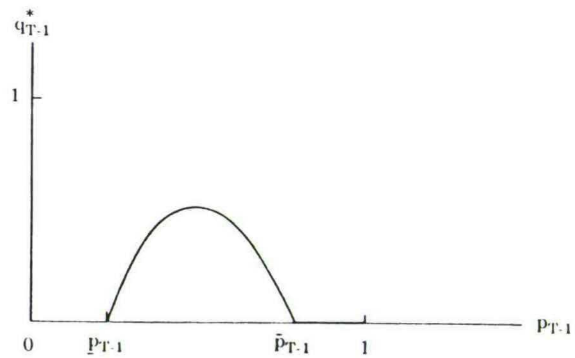

Figure 7b: Relationship between the optimum mixing (q^*_{T-1}) and the initial reputation (p_{T-1})

If the wet government is randomising in T-1, the equality

$$p_T(x_{T-1} = 0) - p_T(x_{T-1} = c) = \frac{1}{2(2\pi-1)} \tag{3.6}$$

must be satisfied, where $p_T(x_{T-1} = 0)$ and $p_T(x_{T-1} = c)$, are determined by equations (3.1a) and (3.1b) above. The left-hand side of (3.6) is a decreasing function of q_{T-1} for $q_{T-1} \in [0,1]$ and $p_{T-1} \in (0,1)$. At $q_{T-1} = 1$, it equals zero, and it reaches a maximum at $q_{T-1} = 0$. Define the LHS of (3.6) as $F(p_{T-1}, q_{T-1}, \pi)$, so we have

$$F(p_{T-1}, q_{T-1}, \pi) \equiv p_T(x_{T-1} = 0) - p_T(x_{T-1} = c). \tag{3.7}$$

$F(.)$ is decreasing in q_{T-1}. $F(0, q_{T-1}, \pi) = 0$, $F(1, q_{T-1}, \pi) = 0$, and $F(p_{T-1}, q_{T-1}, \pi) > 0$ for $0 < p_{T-1} < 1$ and $q_{T-1} < 1$. Thus $F(.)$ can be illustrated as in Figure 8 where each curve in (p_{T-1}, F) space is drawn for a given value of q_{T-1}, and successively higher curves correspond to lower values of q_{T-1}. In the diagram, randomisation occurs for values of p_{T-1} in the interval $(\underline{p}_{T-1}, \bar{p}_{T-1})$. q_{T-1} at first rises and then falls as p_{T-1} rises inside the interval. Outside the interval, the wet government always plays 'intend inflation' with probability one.

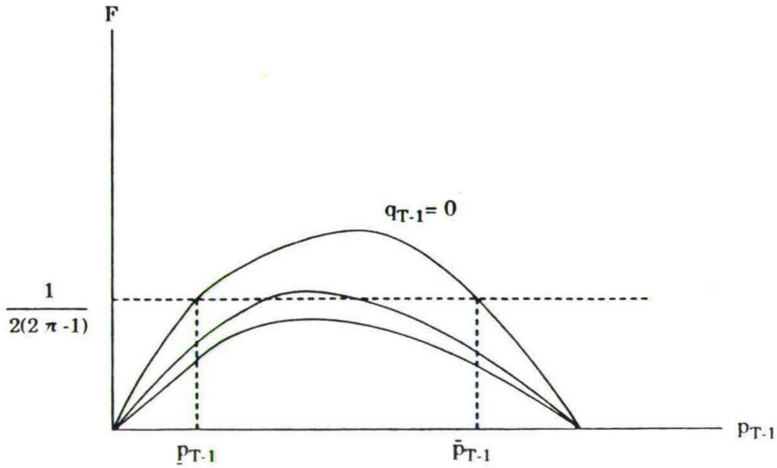

Figure 8: Initial reputation (p_{T-1}) and the difference (F) between posterior reputations in the event of zero inflation and in the event of high inflation

The reputation of the government in period T, $p_T(x_{T-1} = 0)$ or $p_T(x_{T-1} = c)$, is an increasing function of p_{T-1}. This is clearly true for $p_{T-1} \leq \underline{p}_{T-1}$ and $p_{T-1} \geq \bar{p}_{T-1}$, since for these values the optimal $q_{T-1} = 0$ and (3.1a) and (3.1b) immediately indicate that this is so. For $\underline{p}_{T-1} < p_{T-1} < \bar{p}_{T-1}$ this assertion may be proved as follows.

Define

$$F^o(p_{T-1}, q_{T-1}, \pi) \equiv p_T(x_{T-1} = 0)$$

and

$$F^c(p_{T-1}, q_{T-1}, \pi) \equiv p_T(x_{T-1} = c) \tag{3.8}$$

from (3.1a) and (3.1b). Then

$$\frac{dp_T(x_{T-1} = 0)}{dp_{T-1}} = \frac{\partial F^o}{\partial p_{T-1}} + \frac{\partial F^o}{\partial p_{T-1}} \frac{dq_{T-1}}{dp_{T-1}} \tag{3.9}$$

and

$$\frac{dp_T(x_{T-1} = 0)}{dp_{T-1}} = \frac{\partial F^c}{\partial p_{T-1}} + \frac{\partial F^c}{\partial q_{T-1}} \frac{dq_{T-1}}{dp_{T-1}}. \tag{3.10}$$

Since $\dfrac{dp_T(x_{T-1} = 0)}{dp_{T-1}} = \dfrac{dp_T(x_{T-1} = c)}{dp_{T-1}}$ when (3.6) is satisfied, and the government is randomising,

$$\frac{dq_{T-1}}{dp_{T-1}} = -\left[\frac{\partial F^o/\partial p_{T-1} - \partial F^c/\partial p_{T-1}}{\partial F^o/\partial q_{T-1} - \partial F^c/\partial q_{T-1}}\right]. \tag{3.11}$$

Substituting (3.11) into (3.9) gives

$$\frac{dp_T(x_{T-1} = 0)}{dp_{T-1}} = \frac{\partial F^o}{\partial p_{T-1}} - \frac{\partial F^0/\partial p_{T-1} - \partial F^c/\partial p_{T-1}}{1 - F^c_q/F^o_q}. \tag{3.12}$$

Since $F_q^c > 0$ and $F_q^o < 0$, the denominator $(1-F_q^c/F_q^o)$ is greater than one, and clearly the whole RHS in (3.12) is positive. Applying the same treatment to (3.10) confirms that $\frac{dp_T}{dp_{T-1}}(x_{T-1} = c)$ is also positive.

Thus an improvement in reputation at the start of period T-1 results in an improvement in the conditional reputations in period T.

To take the analysis beyond the penultimate period of the game T-1, it is necessary to compute the value function for the government with reputation p_{T-1} entering period T-1, $V(T-1,p_{T-1})$. Whereas for period T, the corresponding function $V(T,p_T)$ is a linear function of p_T, it does not appear to be linear for T-1, and indeed it is not clear that the value function will be monotonically increasing in the reputation p_{T-1}. The reason is as follows:

$$V_1(T-1,p_{T-1}) = \max_{q_{T-1}} W_{T-1},$$

where W_{T-1} is defined in (3.3) above. Expectations of inflation in T-1 are given by (3.2) above for t = T-1.

The optimal choice of q_{T-1} is non-monotonic in p_{T-1}: there may be a range over which q_{T-1} is falling in p_{T-1}. Over this range x^e_{T-1} is rising in p_{T-1}. Consequently, as (3.3) indicates, the effect of the improved reputation in period T may be offset by the effect of higher inflation expectations in T-1. Thus it is not clear that the value function at T-1 is monotonic in reputation T-1, and it does not seem straightforward to extend this analysis beyond two periods.

4. Conclusions

This paper has attempted a number of generalisations of simple models of macroeconomic policy with incomplete information, with the purpose of finding out how sensitive their implications are to changes in assumptions. The basic result of the simple analysis (Backus and Driffill 1985a; Barro 1986) is that the reputation effect can constrain a government to mimic the behaviour of a government which is committed to a zero-inflation strategy, so long as the government has a sufficiently long time-horizon or a sufficiently long period of time remaining in office.

In Section 1 of the paper that simple analysis is extended, following Vickers (1986) to allow for a weaker form of uncertainty about the government's preferences, namely the private sector is unsure about the weight the government places on employment: it may be higher or lower, but it is not zero. It then appears that if the two possible types are relatively similar, they take actions at the start of the game which distinguish the true type of government. The government which is less concerned about employment and output initially signals its type in instituting an inflation rate which is so low (though positive) that the more concerned government would not want to imitate it. If, however, the two possible types of government are relatively dissimilar, a pooling equilibrium is possible in which both types initially institute a zero inflation rate and the true type is only revealed in the last period of the game.

Thus this result supports the Backus/Driffill result. It appears that it is not necessary for the public to believe that the government may be 100 per cent committed to zero inflation for the reputational constraint to induce zero inflation, only that the government may care little enough about output.

The result may appear paradoxical in that separation occurs for relatively similar government types and not for relatively dissimilar types. The reason for this is that the less a government cares about employment, the less 'budgeable' it is. That is to say that it is less willing to deviate from its preferred inflation rate in exchange for a given reduction in the private sector's inflation expectation in the next period. Hence it may be unwilling to set an inflation rate low enough to prevent the more caring government from imitating its action.

Zero inflation emerges in pooling equilibria even when both possible government types have an inflationary bias because, in a pooling equilibrium, the actual and expected inflation rates in the first period are equal, for all types of government, and, when there are no inflationary surprises, all types of government prefer zero inflation to any other. The private sector is modelled formally as being indifferent among alternative fully anticipated inflation rates. Consequently a pooling outcome with zero inflation dominates other pooling equilibria.

Section 2 of the paper extends the same analysis to allow for any possible type of government within some range. In this case, the separating equilibrium applies over a narrower range of types of government, since for each type of government, the adjacent type (with a lower c-value) chooses a low enough inflation rate to separate itself from the next higher type of government. Zero inflation is played by a government with a relatively high preference for employment and inflation. No type of government uses a less-than-zero inflation rate in equilibrium to identify its type. If the range of possible types is wider than the pure separating equilibrium can accommodate, a hybrid pooling/separating equilibrium may emerge with the most inflationary governments separated, but the least inflationary governments pooled. Thus this is an example where, although there is an infinity of possible government types, and an infinity of actions, each type does not take a different action in order to identify itself uniquely. Only the most inflationary governments are exposed.

In Section 3 the simple Backus/Driffill model is extended in a completely different direction. Exogenous noise is introduced into the model, so that the appearance of some inflation is no longer incontrovertible evidence of an uncommitted or inflationary government. It may simply be a random occurrence when the government has, with imperfect controls, attempted to achieve zero inflation. Under these circumstances the incentive for the uncommitted government to imitate the one committed to zero inflation is weakened considerably. In the penultimate stage of the game, an uncommitted government with either a very poor or a very good reputation will intend to cause inflation, and a government with a middling reputation will mix inflating and not inflating, but no uncommitted government will intend zero inflation with certainty.

The simple change to the model made by adding uncertainty about the effects of policy complicates the analysis considerably. Only a two-period game has been analysed in this paper.

It appears from the tentative analysis in this paper that the reputational constraints on macroeconomic policy remain effective when the private sector's beliefs about possible government types are generalised, but imperfect monitoring of government actions weakens those constraints considerably.

Note

* This paper was written while the author was Houblon-Norman Research Fellow at the Bank of England. The work reflects views solely of the author and not necessarily of the Bank. The paper has benefited from comments received in seminars at Aarhus, Bristol, Exeter, Hull and LSE.

References

Backus, D.K. and E.J. Driffill, 'Inflation and reputation', American Economic Review, vol. 75, no. 3, 1985a, pp. 530-538.

Backus, D.K. and E.J. Driffill, 'Rational expectations and policy credibility following a change in regime', Review of Economic Studies, vol. LII, 1985b, pp. 211-221.

Barro, Robert J., 'Reputation in a model of monetary policy with incomplete information', Journal of Monetary Economics, vol. 17, 1986, pp. 3-20.

Canzoneri, M.B., 'Monetary policy games and the role of private information', American Economic Review, vol. 75, no. 5, December 1985, pp. 1056-1070.

Cukierman, A. and A.H. Meltzer, 'A theory of ambiguity, credibility and inflation under discretion and asymmetric information', Econometrica, vol. 54, no. 5, September 1986, pp. 1099-1128.

Kreps, D. and R. Wilson, 'Reputation and incomplete information', Journal of Economic Theory, vol. 27, 1982, pp. 253-279.

Rogoff, K.D., 'Reputational constraints on monetary policy', (mimeo), July 1986.

Rubinstein, A., 'An optimal conviction policy for offences that may have been committed by accident', in S.J. Brams, A. Scholter and G. Schwodiauer (eds), Applied Game Theory, Physica-Verlag, Wurzburg, 1979.

Söderström, H.T., 'Union militancy, external shocks, and the accommodation dilemma', Scandinavian Journal of Economics, vol. 87, no. 2, 1985, pp. 335-351.

Vickers, J., 'Signalling in a model of monetary policy with incomplete information', Oxford Economic Papers, vol. 38, 1986, pp. 443-455.

Appendix

In this Appendix we investigate whether, for the cases in Section 1 of the paper where $c_1 < c_2/2$, the proposed pooling equilibria dominate the separating equilibrium, when both exist.

In the separating equilibrium (1.3a) above, the equilibrium strategies are

$$
\begin{aligned}
&x_{T-1(1)} = k_B \\
&x_{T-1(2)} = c_2 \\
&x^e_{T-1} = pk_B + (1-p)c_2 \\
&x_{T(1)} = c_1 \\
&x_{T(2)} = c_2 \\
&x^e_T = \begin{cases} c_1 \text{ if } x_{T-1} \le k_B \\ c_2 \text{ if } x_{T-1} > k_B. \end{cases}
\end{aligned}
\tag{A.1}
$$

Thus the utilities over the whole game are

$$
\begin{aligned}
U^s_1 &= U_1(x^e_{T-1}, x_{T-1}, x^e_T) = -k^2_B/2 + c_1(1-p)(k_B-c_2) - c^2_1/2 \\
U^s_2 &= U_2(x^e_{T-1}, x_{T-1}, x^e_T) = -c^2_2 + c_2p(c_2-k_B).
\end{aligned}
\tag{A.2}
$$

In the pure pooling equilibrium which obtains when $c_1 < c_2/2$ and $\bar{c} \le c_2/2$, we have in equilibrium

$$
\begin{aligned}
&x_{T-1(1)} = x_{T-1(2)} = x^e_{T-1} = 0 \\
&x^e_T = \bar{c} = pc_1 + (1-p)c_2 \\
&x_{T(1)} = c_1 \\
&x_{T(2)} = c_2
\end{aligned}
\tag{A.3}
$$

and the utilities are correspondingly

$$
\begin{aligned}
U_1^p &= U_1(x_{T-1}^e, x_{T-1}, x_T^e) = -c_1^2/2 + c_1(c_1-c_2)(1-p) \\
U_2^p &= U_2(x_{T-1}^e, x_{T-1}, x_T^e) = -c_2^2/2 + c_2 p(c_2-c_1).
\end{aligned}
\tag{A.4}
$$

It is clear that $U_1^p \geq U_1^s$ and $U_2^p \geq U_2^s$ and thus that the pooling equilibrium dominates the separating equilibrium for both government types.

(For the case of the type-2 government, subtraction of (A.2) from (A.4) yields an expression which gives on rearrangement

$$U_2^p - U_2^s = c_2(c_2/2-c_1+k_B) - c_2(1-p)(k_B-c_1).$$

The second term, $c_2(1-p)(k_B-c_1)$ is clearly negative. The first term is positive because $c_2/2 - c_1 \geq -k_B$ when $c_1 \leq c_2/2$, as reference to Figure 3 shows. In Figure 3, U_2 cuts the vertical axis at 45°, but the U_2 curve is strictly concave, thus $c_2/2 - c_1 \geq -k_B$.)

If private sector expectational errors are examined, it is found that the sum of expected squared errors over the two periods are smaller in the pooling equilibrium. Thus all players in the game prefer the pooling equilibrium to the separating equilibrium in this case.

Now turn to the case where $c_1 < c_2/2$ but $\bar{c} > c_2/2$ and there is a mixed pooling solution. The equilibrium strategies are now

$$
\begin{aligned}
x_{T-1(1)} &= 0 \\
x_{T-1(2)} &= \begin{cases} 0 \text{ with probability } q \\ c_2 \text{ with probability } 1-q \end{cases} \\
x_{T-1}^e &= c_2(1-p)(1-q) \\
x_{T(1)} &= c_1 \\
x_{T(2)} &= c_2 \\
x_T^e &= c_2/2 = \hat{p}c_1 + (1-\hat{p})c_2 \text{ if } x_{T-1} = 0 \\
&\text{and } c_2 \text{ if } x_{T-1} > 0.
\end{aligned}
\tag{A.5}
$$

where $\hat{p} = \frac{p}{p + (1-p)q}$.
Now the utilities are

$$U_1^M = -c_1c_2(1-p)(1-q) - c_1^2/2 + c_1(c_1-c_2/2)$$
$$\tag{A.6}$$
$$U_2^M = -c_2^2(1-p)(1-q).$$

It can be shown that $U_2^M - U_2^s \geq 0$ and $U_1^M - U_1^s \geq 0$. Subtracting (A.2) from (A.6) for type 2 gives

$$U_2^M - U_2^s = c_2^2(1-p)q + c_2pk_B. \tag{A.7}$$

By (1.11) we deduce that

$$\frac{p}{p + (1-p)q} = \frac{c_2/2}{c_2 - c_1}$$

and hence

$$p + (1-p)q = 2p\,\frac{c_2 - c_1}{c_2}. \tag{A.8}$$

Substituting for $(1-p)q$ in (A.6) gives

$$U_2^M - U_2^s = c_2^2(2p\,\frac{c_2-c_1}{c_2} - p) + c_2pk_B$$

$$= c_2p(c_2-2c_1+k_B).$$

As it was shown above that $k_B \geq -c_2/2 + c_1$, it is clear that $U_2^M - U_2^s \geq 0$.

Consider now $U_1^M - U_1^s$. Subtracting (A.2) from (A.6) gives

$$U_1^M - U_1^s = -c_1c_2(1-p)(1-q) + c_1(c_1-c_2/2)$$

$$+ k_B^2/2 - c_1(1-p)(k_B-c_2)$$

and we use (A.8) again to substitute out q yielding

$$U_1^M - U_1^S = -c_1((1-2p)c_2+2pc_1) + c_1(c_1-c_2/2) + k_B^2/2 - c_1(1-p)(k_B-c_1)$$

$$= -c_1(1-2p)(c_2/2-c_1) - c_1k_B(1-p) + k_B^2/2$$

$$\geq -c_1(c_2/2-c_1) - c_1k_B + k_B^2/2. \tag{A.9}$$

From the definition (1.3) of k_B we derive the condition

$$k_B^2/2 - c_2k_B - c_2^2/2 + c_1c_2 = 0$$

which, when subtracted from (A.9), yields

$$U_1^M - U_1^S \geq (c_1-c_2)(-k_B+c_1-c_2/2) \geq 0,$$

since both bracketed terms, (c_1-c_2) and $(-k_B+c_1-c_2/2)$, are negative.

Hence both types of government prefer the mixed pooling equilibrium to separating when $c_1 \leq c_2/2$ and $\bar{c} \geq c_2/2$. However, it does not appear to be the case that the private sector prefers the mixed pooling equilibrium to the separating equilibrium. In the mixed-pooling outcome, the private sector makes an error of $c_2(1-p)(1-q)$ with probability $p + (1-p)q$, and $c_2(1-(1-p)(1-q))$ with probability $(1-p)(1-q)$ in the first period. And in the second period, it makes no error if $x_{T-1} = c_2$ was observed in the first. But if $x_{T-1} = 0$ was observed in the first period, then it makes an error of $c_2 - c_2/2$ with probability $(1-\hat{p})$ and $(c_1-c_2/2)$ with probability $\hat{p}$ in the second period.

The expected value of squared expectational errors in the mixed-pooling case ($= -U_p^M$) gives

$$-U_p^M = 2p(c_2-c_1)(c_2-2p(c_2-c_1)) + p(c_2-c_1)(c_2/2-c_1)$$

which compares with the value in the separating equilibrium of

$$-U_p^S = p(1-p)(k_B-c_2)^2.$$

Thus this equilibrium may not dominate the separating equilibrium, although it is not dominated by it, and so the separating equilibrium cannot be ruled out on grounds of equilibrium dominance in the case $c_1 \leq c_2/2$, $\bar{c}(= c_1p+(1-p)c_2) > c_2/2$.

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